Correlation Primer[1]

August 6, 2004

Table of Contents

I. Introduction

Fixed income professionals are focusing intently on "correlation" as a factor that drives the value of their portfolios. Recent waves of bond defaults in certain sectors have prompted heightened attention on correlation. Correlation arguably has become most important in the areas of credit derivatives, such as collateralized debt obligations (CDOs) and baskets of credit default swaps (CDS). In those sectors, some market participants recently have begun to trade correlation.

In this paper, we provide a brief tour of basic correlation concepts and their applications. We explain the use of a simple "copula" approach for modeling correlation of credit risk. Lastly, we illustrate how we can price certain structured credit products using the copula technique.

II. Why Do We Care About Correlation?

Correlation is important in the credit markets. It affects the likelihood of extreme outcomes in a credit portfolio. Therefore, it plays a central role in pricing structured credit products, such as tranches of CDOs, traded CDS indices and first-to-default baskets. Intuitively, when correlation among credits in a portfolio is high, credits are likely to default together (but survive together, too). In other words, defaults in the portfolio would cluster.

Correlation between two variables generally is expressed by a "correlation coefficient," sometimes denoted by the Greek letter Rho (ρ). A correlation coefficient ranges between -1 and +1. When a correlation coefficient is +1, it reflects "perfect positive correlation," and the two variables always move in the same direction. On the other hand, if the correlation coefficient is -1, the variables always move in opposite directions – "perfect negative correlation."

Correlation is closely related to the idea of "diversification." "Diversification" describes a strategy of reducing risk in a portfolio by combining many different assets together. Diversification *presumes* that movements in the values of individual assets somewhat offset each other. More formally, a diversification strategy relies on the assumption that movements in asset values are not perfectly

[1] This is the second piece of a series on basic concepts of structured credit products. For the basics of single-name credit default swaps (CDS), please see *CDS Primer*, Nomura Fixed Income Research, 12 May 2004.

Contacts:

Michiko Whetten
(212) 667-2338
mwhetten@us.nomura.com

Mark Adelson
(212) 667-2337
madelson@us.nomura.com

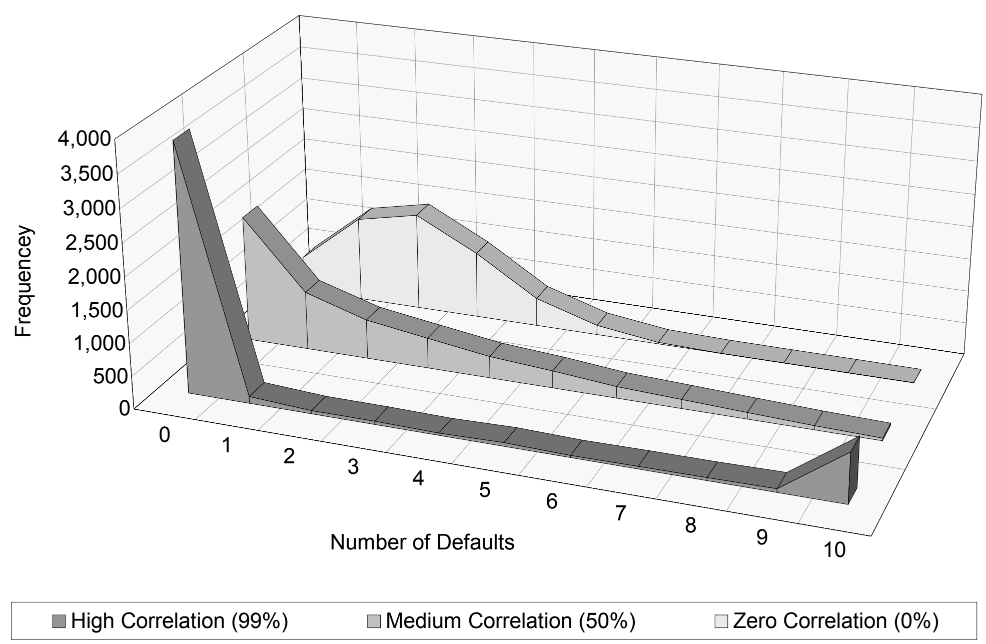
Nomura Securities International, Inc.
Two World Financial Center
Building B
New York, NY 10281-1198
Fax: (212) 667-1046

correlated. In fact, diversification can achieve the greatest reduction in risk when assets display negative correlation. For example, suppose a portfolio consists of two assets of equal value. If returns on the two assets are perfectly negatively correlated, a decline in the value of one asset would be exactly offset by an increase in the value of the other.

In the real world, there is usually some degree of positive correlation among the credit risk of individual assets in a portfolio. The actual degree of correlation strongly influences the distribution of outcomes that the portfolio may experience. Slicing the portfolio into several "tranches" of credit priority magnifies the importance of correlation because the degree of correlation affects the value of different tranches differently.

Consider the most junior and most senior tranches of a CDO. All else being equal, if the credit risk among the underlying assets is strongly correlated, there is a higher likelihood of either very few or very many of the assets defaulting. Conversely, if correlation is weak, the likelihood of extreme outcomes is low, and the likelihood of an intermediate number of defaults is higher. Consider a hypothetical portfolio of 10 equally sized assets, where each asset has a 20% probability of default. If the risk among the assets is uncorrelated (*i.e.*, ρ=0), the most likely outcome would be two defaults, and the chance of more than five assets defaulting would be extremely small. However, if the risk among the assets is correlated to a significant degree, the odds of either (1) no defaults or (2) more than five defaults increase substantially. Graph 1 illustrates this with results of a simulation.

Graph 1: Frequency of Portfolio Default and Tranche Losses
(Simulation Results)



Based on simulations with 5,000 iterations assuming a 20% probability of default.
Source: Nomura

Graph 1 shows the frequencies of defaults in three hypothetical portfolios of 10 assets with zero (ρ=0%), medium (ρ=50%), and high (ρ=99%) levels of correlation, based on the results of a Monte Carlo simulation. The simulation results show that the distribution of defaults can vary greatly depending on the level of correlation. In the graph, the default frequency for the zero-correlation case looks somewhat like a bell-shaped curve. In contrast, the medium- and high-correlation cases exhibit downward sloping and "U"-shaped curves, respectively. The high-correlation portfolio has the highest frequency for zero defaults, while the low-correlation portfolio usually has a modest number of defaults. Also notable are the differences in the frequencies of large numbers of defaults. The right end of the graph shows that the high-correlation portfolio has a much higher frequency of all assets defaulting, compared to the zero- and the medium-correlation portfolios.

Returning to the hypothetical CDO backed by the 10 assets, assume that the equity tranche (*i.e.*, the most subordinated tranche) represents 10% of the deal. Assume further that each asset pays nothing if it defaults. Thus, the equity tranche would be wiped out if any of the 10 assets defaults. On the other hand, if none of the assets defaults, the equity tranche receives its cash flow. Given such a structure, the value of the equity tranche would be highest in the case of the high-correlation portfolio. The reason is that the high-correlation portfolio has the greatest likelihood of experiencing zero defaults. Moreover, after one default, it makes no difference to the equity tranche whether there are additional defaults because the first default wipes out the tranche.

Now consider the CDO's senior tranche. Suppose that the senior tranche accounts for 60% of the deal and, therefore, does not incur losses until there are more than four defaults. The situation for the senior tranche is exactly opposite to that of the equity tranche. When the portfolio's correlation is zero, there is only a slight possibility that many defaults occur and cause losses for the senior tranche. On the other hand, a high correlation means an increased possibility of a large number of defaults, where all junior tranches are exhausted and losses reach the senior tranche. Accordingly, an increase in correlation decreases the value of the senior tranche.

Hence, correlation affects equity and senior tranches in opposite directions. Until last year, correlation in structured credit products, such as CDO tranches or first-to-default baskets, was analyzed and priced mostly at individual banks and dealers. The introduction of tranche trading on the two CDS indices, Trac-X℠ and iBoxx℠,[2] changed that. Tranche trading in the indices put correlation in the spotlight and drove development of common valuation methods. The recent merger of the formerly competing index families should boost activity in CDS index trading and sustain the market's focus on correlation as key issue.

III. Analyzing a Portfolio of Credit Risk

Market participants usually use Monte Carlo simulations to analyze the impact of correlation in portfolios of credit risk. When one needs to take correlation into account (*i.e.* variables are not independent), Monte Carlo simulations allow greater flexibility than non-simulation techniques. This section describes the use of "default time" as a tool for modeling the credit risk on individual credits and then illustrates the use of a "copula" approach for analyzing correlated risk on a portfolio of credits through a simulation. After simulating correlated defaults, we can value an individual tranche of a portfolio tranche and a "first-to-default" basket of credit risk. Discussion of more advanced versions of copula models is beyond the scope of this report, but we provide actual simulation results and some mathematical basics of the copula approach in the Technical Appendix.

A. Modeling Correlated Defaults

1. Modeling Default Time of Individual Credits

One popular way to model correlated default risks is to focus on correlation between “default times” (*i.e.* when assets or companies default). An asset's "default curve" plots the asset's default probability over time. A default curve permits calculation of the likelihood that the asset will survive (*i.e.*, not default) for a specified period. For example, if an asset has a constant default probability of 5% per year, it means that the likelihood that the asset will survive for one year is 95% (=100% - 5%). The survival probability goes down over a longer time horizon: over five years the asset's survival probability is 77.38% (=0.95^5). Accordingly, the asset's default probability over five years is 22.62% (= 100% - 77.38%).

Within the context of a simulation, we reverse the process. Our goal is to determine whether the asset defaults over a certain time horizon, and we start with the survival probability and arrive at an

[2] The two CDS index groups merged in April 2004. The new indices are called CDX in North America and iTraxx in Europe, Asia and Australia.

asset's default time. To generate a simulated default time, we use a uniform random variable[3] having the range 0 to 1 and the asset's survival curve. We treat the random variable as the survival probability and find the corresponding default time on the asset's survival curve. For example, a simulated random value of 0.5 means that the survival probability to a specific time point is 50%. Continuing with the above example, we find the simulated default time using the below equation:

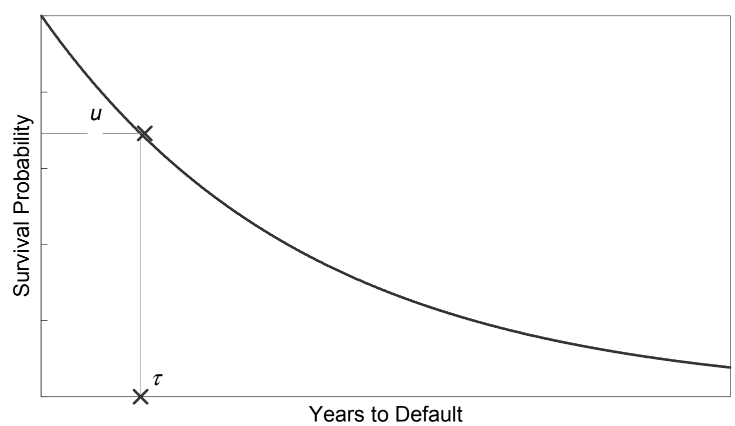
$$95\%^{\wedge}\tau = u$$

where:

95%	=	the annual survival probability
τ	=	the default time, expressed in years
u	=	simulated uniform random variable

Replacing u_i with 0.5 solving for τ, we get τ = 13.513 years. In general, when there are multiple assets, "default correlation" refers to the tendency of assets to default together over a particular time horizon. Credit risk simulations often capture correlation by linking the default times of different assets.[4]

Graph 2: The Mapping of Survival Probability to Default Time



2. What is A Copula?

The most commonly used method for modeling and simulating correlated default risk is the "copula approach." The word "copula" comes from the Latin word for "connecting" or "linking." In technical terms, a copula function is a *multivariate*, or joint, distribution that links a set of *univariate*, or

[3] A uniform distribution is a random distribution where probability density is constant over a defined range. For example, a uniform distribution with a range of 0 to 1, any number between 0 and 1 is possible with equal likelihood.

[4] This point was discussed in Li, D., *On Default Correlation: A Copula Function Approach*, RiskMetrics Group, 2000. Li originally proposed the application of the copula approach in credit risk modeling.

marginal, distributions.[5] While the marginal distributions describe the default risk of each individual asset, the copula describes relations among individual default risks through a multivariate distribution. The key point is that copula allows us to separately specify the characteristics of individual default risks and the relationship among them. Although it is possible to use a multivariate distribution to directly model individual default risks together, it is usually much easier to use the copula approach when more than two variables need to be modeled.[6]

One popular model for simulating correlated credit risks is the "one-factor Gaussian copula." The term "one factor" refers to using one extra variable to which the performance of each individual asset is linked. The extra variable can be viewed as expressing the condition of the overall economy. The term "Gaussian" refers to the underlying assumption that the correlations among the underlying assets can be expressed with "normal" distributions, even if the individual credit risks themselves are not normally distributed. A further key assumption is that correlation among the underlying assets remain stable over time. For now, it remains unclear whether this assumption fairly reflects reality. However, dropping the assumption arguably complicates the modeling process without necessarily improving predictive power.

3. Simulating Correlated Defaults with a Copula

A Monte Carlo simulation using a copula model involves a few simple steps. Suppose that we have credit exposure to a portfolio of three assets. To use a one-factor Gaussian copula to simulate the portfolio, we would need the following ingredients:

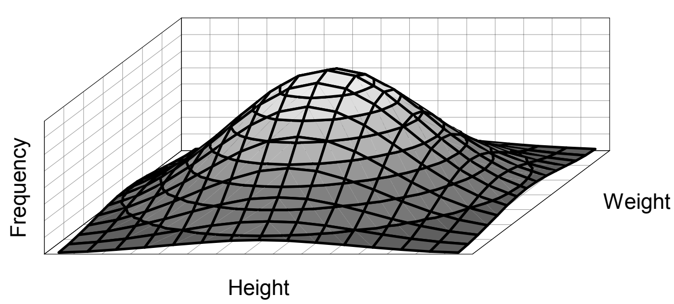
- For each of the three assets, an assumption about asset's risk of default over different time horizons (*i.e.* the asset's default curve and its corresponding survival curve).

- For each of the three assets, an assumption about the level of recovery that would be associated with a default.

- An assumption about the correlation between the credit risk of each asset and the overall economy. Intuitively, correlation describes the degree to which different assets default around the same time. In the simplest case, we would assume a uniform correlation structure, where all the assets have the same correlation of credit risk to the overall economy. In that case, we have one ρ for all calculations.

---

[5] The term "univariate" refers to a distribution that involves just one variable. The term "multivariate" refers to a distribution that involves two or more variables. A multivariate, or joint, distribution describes the behavior of two or more variables. We can visualize the multivariate distribution of two variables with a three-dimensional graph. For example, the following chart shows a hypothetical joint distribution of height and weight for a group of people:



[6] For example, modeling default risk of two credits jointly is not very difficult. In fact, if we assume normal distributions for the individual variables, there is no difference between using a copula and modeling the default risk directly via a multivariate normal distribution.

Each iteration of the copula-based simulation involves the following four steps:

The first step is to generate a normally distributed random variable for each of the three assets, plus a fourth one for the overall economy. Let the random variable for the three assets be ε_1, ε_2, and ε_3, respectively. Let the random variable for the overall economy be y.

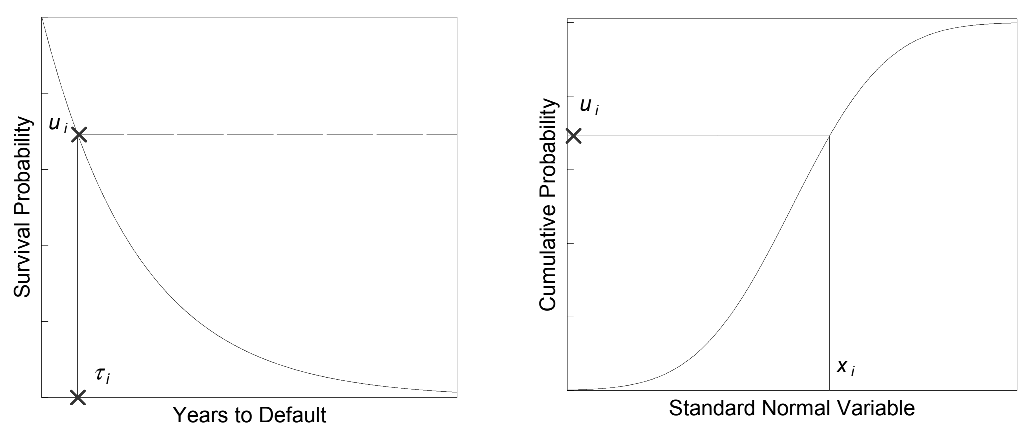
The second step is to use the ε_i variables, together with the correlation coefficient, to produce a group of three *correlated* normally distributed random variables: x_1, x_2, and x_3. Assuming a uniform and stable correlation structure, we can use the following formula to produce the x_i variables:

$$x_i = \sqrt{\rho} y + \sqrt{1-\rho}\varepsilon_i$$

The third step is to produce corresponding set of variables in the range of 0 to 1. Each of the x_i variables is normally distributed and we can use the cumulative distribution function[7] of the standard normal distribution to convert it into a corresponding value in the 0-1 range. We denote those values as u_1, u_2, and u_3. The use of the normal distribution in this step is why the process is called a "Gaussian copula." (The chart below on the right illustrates this step.)

The fourth and final step of the process is to convert the u_i values into default times for each of the assets. The easiest way to do this is to use the survival curves. The survival curves allow us to map each u_i variable to a corresponding default time (τ_i). The default time for a particular asset tells whether (and when) the asset defaulted in that iteration of the simulation. (The chart below on the left illustrates how to map each u_i to the corresponding τ_i.)

Graph 3: The Mapping of Simulated Correlated Random Variables to Default Time



Source: Nomura

B. Valuing Tranched Transactions

Once we have simulation results, how do we value a tranche trade? A Monte Carlo simulation generates a large number of randomly generated scenarios of correlated defaults. Based on the simulation results, we can calculate expected losses and, hence, breakeven spreads of tranches. Let's consider a portfolio of CDS consisting of five reference entities.[8] We assume that each reference entity has a notional amount of \$100, an annual default probability of 5%, and a fixed

[7] A cumulative distribution function gives the probability that a random variable falls below a certain value.

[8] Our discussion assumes a CDS-like structure, with "reference entities" instead of assets. The analysis can be applied to cash CDOs as well.

recovery rate of 45%. We assume 1% per year for the discount rate. We also assume that the time horizon is five years. A simulation path might look like the following table:

Table 1: A Sample Simulated Path of Default Times		
	Default Time	Default before maturity (5 years)
Credit 1	2.5 years	Yes
Credit 2	4.5 years	Yes
Credit 3	8.0 years	No
Credit 4	10.0 years	No
Credit 5	11.0 years	No

In this simulation path, default times of two reference entities are less than five years, indicating that the two defaults occurred during the term of the portfolio. Given the recovery rate of 45%, the loss amount is $55 for each of the two defaulting credits. Below, we illustrate how we can value two simple structured credit products; loss tranches and n^{th}-to-default baskets.

1. Valuing a Loss Tranche

Percentage loss tranches are constructed so that one tranche absorbs losses until the total loss reaches the size of the tranche. After the first-loss tranche is exhausted, the next junior tranche starts to absorb losses until it is exhausted. In other words, a loss tranche has an "attachment point" and a "detachment point," characterizing the seniority and size of the tranche. For example, a tranche with an attachment point of 0% and a detachment point of 20% bears exposure to the first $100 of losses in a portfolio of $500.

For example, Table 2 shows the same simulated path we discussed above. In the column for the 0%-20% tranche, we can see that the tranche suffers a loss in the third year (2.5 years) of $55, as the first default occurs. Further, the same tranche suffers another $45 in the fifth year (4.5 years) as the second default occurs in the portfolio. Since the tranche size is $100, the second default wipes out the 0%-20% tranche ($55+$45=$100) and losses reach the next tranche (the 20%-40% tranche). Accordingly, this particular path results in a $100 loss (with a present value of $96.66) to the 0%-20% tranche and a $10 loss (PV of $9.56) to the 20%-40% tranche.

Table 2: Losses to % Tranches - the Sample Path					
Year / Tranche	0%-20%	20%-40%	40%-60%	60%-80%	80%-100%
1	0	0	0	0	0
2	0	0	0	0	0
3	$55	0	0	0	0
4		0	0	0	0
5	$45	10	0	0	0
Total Loss	$100	10	0	0	0
Loss PV	$96.662	$9.560	0	0	0

Once we calculate tranche losses for one path, we can calculate the breakeven spread based on the present value of the tranche losses so that the present values of losses equal to the present value of premium paid by the protection buyer.[9] Table 3 shows actual simulation results with 10,000 iterations

[9] In order to calculate a breakeven spread, we need to get the tranche loss and the average notional amount for the tranche. We calculate the average notional amount to take into account that the tranche's notional amount is reduced when defaults occur. For example, if just one default occurs at 2.5 years for the 0%-20% tranche in the above example, the tranche's notional is reduced by $55 to $45. After that point, the protection buyer pays an annual premium to the seller based on the reduced notional amount of $45, not $100. In this particular case, the notional amount is $100 for Year 1, $100 for Year 2, $72.5 for Year 3, $45 for Year 4 and $45 for Year 5. (Note that the third year starts with a notional amount of $100 but ends with $45.) Hence, the present value of the average notional is about $71, at a discount rate of 1% per year. We aggregate all the simulation paths and calculate average present values of the average notional amounts and the tranche losses. Finally, we divide the average present value of annual losses in by the average notional amount, also in present value terms, to arrive at a breakeven spread for the tranche.

and correlation of 0.5. In Table 3, the average present value of tranche losses is $41.88 and the breakeven spread is 1,129 bps.

Table 3: Combined Simulation Results For the 0%-20% Tranche			
Simulation path	# of portfolio defaults	PV of Tranche loss	Breakeven Spread
1	0	0	
2	0	0	
3	1	$53.642	
⋮	⋮	⋮	
10,000		0	
Average	1.112	$41.876	1129 bps

2. Valuing Nth-to-Default Basket Trade

In a first-to-default (FTD) basket, the buyer of protection receives the par amount minus the recovery if one or more reference entities default before maturity. On the other hand, the protection seller receives periodic premium payments (spread) until one or more defaults occur. Likewise, the buyer of a second-to-default protection receives a payment if two or more defaults occur, while the protection seller continues to receive spreads until two or more reference entities default, and so on. Unlike in loss tranches, each default knocks out one tranche at a time, sequentially moving from junior to senior ones.

Table 4: The Nth-to-Default Basket - A Simulated Path of Default Losses						
Year	Credit 1	Credit 2	Credit 3	Credit 4	Credit 5	
1	0	0	0	0	0	
2	0	0	0	0	0	
3	$55	0	0	0	0	<= 1st-to-default is knocked out.
4		0	0	0	0	
5		$55	0	0	0	<= 2nd-to-default is knocked out.

Table 4 shows the same sample path as before. The first default occurs in the third year (2.5 years) and the second default occurs in the fifth (4.5 years) year. Accordingly, the first-to-default and the second-to-default protections cease to exist after the third year and the fifth year, respectively.

Table 5: Losses For the Nth-to-Default					
Year	FTD	2nd-to-default	3rd-to-default	4th-to-default	5th-to-default
1	0	0	0	0	0
2	0	0	0	0	0
3	$55	0	0	0	0
4		0	0	0	0
5		$55	0	0	0
Total Loss	$55	$55	0	0	0
Loss PV	$53.642	$52.580	0	0	0

As shown in Table 5, this particular path dictates very large losses for the first-to-default tranche ($53.64) and the second-to-default tranche ($52.58). However, because two out of the five credits defaulting within 5 years is a highly unlikely event, such a scenario is associated with a small probability. As before, we collect all simulated paths and calculate the average present value (PV) of losses to the first-to-default (FTD) tranche in order to arrive at the breakeven spread for that tranche.

Table 6: Combined Simulation Results For the 1st-to-Default Basket (ρ = 0.5)			
Simulation path	# of portfolio defaults	PV of Tranche loss	Breakeven Spread
1	0	0	
2	0	0	
3	1	$53.669	
⋮	⋮	⋮	
10,000		0	
Average	0.112	$28.486	849 bps

On a 10,000-path simulation with correlation of 0.5, the first-to-default tranche is expected to suffer an average loss of $28.49 in present value and the breakeven spread is 849 bps. In a similar manner, we can calculate breakeven spreads for more senior tranches. See the Technical Appendix for details of our simulation results.

C. Trading a Portfolio of Credits

1. Introduction of The CDS Indices

In April 2004, the backers of the two rival credit derivatives index families, DJ Trac-x and iBoxx, reached an agreement to create a combined credit derivative index series. In North America, a new credit derivatives index is now called Dow Jones CDX, while the European and Asian CDS indices are Dow Jones iTraxx. The merger is widely welcomed in the credit derivatives market, as it is expected to increase liquidity in the new index and index-related products, such as index tranches and credit index options.

2. The CDS Indices

The CDX index is a static portfolio of 125 equally weighted single-name CDSs. The index is "rolled" into a new contract every six months, and reference entities to be included in the new series are selected through a dealer poll. There are multiple maturities, with the most liquid being the 5- and 10-year maturities. There are also sub-sector indexes for sectors such as financials, consumers, energy, industrials, telecoms & media, and high volatility.

In an unfunded transaction, an investor, or a protection seller, enters into a standardized CDS contract with a market maker, following the 2003 ISDA Credit Derivatives Definitions. The investor receives a quarterly premium, or spread, on the index, unless credit events occur. If a credit event occurs, the investor delivers the cash settlement amount determined by the weighting of the defaulted reference entity and the recovery rate. After a credit event, the notional amount for the transaction declines by the portion of the defaulted credit.[10] While trading an index gives a means of quick diversification for investors, it has also boosted market liquidity and contributed to the rapid growth of the credit derivatives market as a whole.

3. Trading Index Tranches

Most notable in the new generation of CDS index products is tranche trading. The weekly trading volume of the standardized tranches is estimated to be between $500 million and $600 million. There are standard tranches on the DJ CDX NA Index with attachment and detachment points of 0%-3%, 3%-7%, 7%-10%, 10%-15%, and 15%-30%. These are so-called "loss tranches," which we discussed above.

For example, the 7%-10% tranche covers losses up to $30 million on the 125-name index portfolio with the notional amount of $1 billion. Each reference entity has $8 million notional. The 7%-10% tranche suffers no losses until portfolio losses exceed 7% of the whole portfolio, or $70 million. After

[10] As of the time of writing, discussions continue with regard to details of the documentation for the merged index.

that point, any incremental losses would reduce the tranche size dollar-for-dollar. For instance, if another credit defaults, and if we assume a 45% recovery rate, the notional amount of the 7-10% tranche would be reduced by $4.4 million ($8 million x (1-recovery rate)). If the portfolio losses surpass $100 million, however, losses to the 7%-10% tranche are capped at the tranche size of $30 million because the tranche gets wiped out. In other words, the 7%-10% tranche is a mezzanine tranche with 7% subordination.

Table 7: Sample Spreads for Tranched DJ CDX NA IG (July 27, 2004)

Tranche	Typical Rating	Market Spreads	Implied Correlation
0%-3%	n.a.	40.9 pts + 500 bps	23%
3%-7%	BBB+ / BBB	340 bps	3%
7%-10%	AAA / AA+	133 bps	19%
10%-15%	AAA	44 bps	20%
15%-30%	n.a.	15 bps	31%
Underlying DJ CDX		62.25 bps	N.A.

Source: Creditflux, Fitch, Bloomberg

Why would one want to trade an index tranche? First, index tranches allow leveraged positions in a portfolio of very liquid CDS. The 0%-3% tranche allows roughly 33 times leverage while capping the holder's risk at 3%. Generally, the most junior tranche in the index, the 0%-3% tranche, is quoted with "points" to be paid upfront and a fixed premium of 500 bps per annum, instead of variable spreads. This large carry reflects the high risk associated with the "equity" tranche. In contrast, purchasing protection in the senior tranche would provide an efficient way to insure against large downside risk for a relatively low premium. Second, tranche trading facilitates separation of "spread" risk from "default" risk. So, for example, an investor might sell protection in a 0%-20% tranche and might buy protection in an appropriate amount of 10%-15% tranche to hedge against the risk of spread movements. However, in doing so, the investor would be able to maintain the exposure to default risk in the portfolio.

4. What is "Implied Correlation"?

The most important element of tranche trading is "implied correlation." As discussed above, default correlation affects the values of the portfolio tranches, because it drives the shape of the portfolio's loss distribution. In pricing a loss tranche (or an n^{th}-to-default trade), we use default probabilities of individual reference entities *and correlation* as the model inputs. Conversely, given market spreads for portfolio tranches, we can "back out" the implied level of correlation using the same pricing model. Implied correlation, therefore, is a very similar concept to implied volatility in the options market. High correlation increases the value of equity tranches, while low correlation benefits senior tranches. Hence, given market prices, a high implied correlation indicates relative "richness" in junior tranches and relative "cheapness" in the senior tranches, and vice versa.

IV. Technical Appendix

A. Exact Simulation Algorithm

Our simulation algorithm, shown below, is a simple and mechanical process. First, we generate uncorrelated, independent, random variables for each credit as well as one common factor from a standard normal distribution. Second, we transform these uncorrelated variables into correlated ones via a copula function. Then we translate the correlated variables into default time. Finally, we evaluate whether the simulated default time for each credit exceeds the specific time horizon, say 5 years.

The table below shows one simulation path generated using the approach described above:

Table 8: Simulation Steps

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Common normal	Credit	Uncorrelated normal (ε_i)	Correlated normal (x_i)	Correlated uniform (u_i)	1-correlated uniform	Years to default (τ_i)	Default?
-1.1166	I	-1.2240	-1.6551	0.0490	0.9510	0.979	Yes
-1.1166	ii	0.4413	-0.4775	0.3165	0.6835	7.419	No
-1.1166	iii	-0.9347	-1.4505	0.0735	0.9265	1.488	Yes
-1.1166	iv	0.4357	- 0.4815	0.3151	0.6849	7.378	No
-1.1166	v	-2.3653	-2.4621	0.0069	0.9931	0.135	Yes

In column (1), an uncorrelated, independent random number is sampled from the standard normal distribution. This number serves as the common factor, which can be viewed as representing the overall economy.

Similarly, in column (3), uncorrelated random numbers are sampled from the standard normal distribution for each of the companies represented in the portfolio. (Here we have five such entities.)

Then we calculate the correlated normal variable for each credit from the two uncorrelated random numbers, via the one-factor Gaussian copula function. The copula function we use here is a bivariate (*i.e.* with two variables) normal distribution function. In order to generate x_i's which are correlated with each other, plug in the two uncorrelated normal random variables into the following equation:

$$x_i = \sqrt{\rho}y + \sqrt{1-\rho}\varepsilon_i \qquad \text{for } i = 1. \dots, n$$

This relation gives the pair-wise correlation of ρ for any two entities. Assuming ρ = 0.5, the resulting correlated x_i's are in column (4).

In column (5), we convert the correlated normal numbers into uniformly distributed random variable using the cumulative normal function. Subtract this from one, as in column (6). The resulting number is the cumulative "survival" probability, or the probability that the default time falling before a certain point.

Finally, we can calculate the time to default that corresponds to the survival probability in column (6). Using each entity's survival curve, we identify in column (7) the number of years to default that matches the survival probability. In our example we use a flat default probability of 5% per year. Check if default occurred within 5 years, as shown in column (8). Repeat the above steps for each of five reference entities.

The above steps constitute one simulation path. We repeat the whole process for 5,000 or 10,000 repetitions and obtain the average losses and the average notional amount, both in present values.

Based on the simulated default times, calculate the breakeven spread levels that equate the present value of tranche losses and that of premium payments.

B. Our Simulation Results

We ran ten sets of simulations with 10,000 trials for each set. First, we simulated correlated defaults for correlations of 0%, 30%, 50% 70%, and 99%.[11] Then we calculated average losses and spreads for loss tranches of 0%-20% and 40%-100% to demonstrate how correlation affects tranche losses and spreads. We also used the simulated defaults to calculated losses and spreads for nth-to-default tranches. As before, the credit curve is assumed to be flat for all reference entities with a constant default probability of 5% per year. In calculating the breakeven spreads, we assumed 1% per annum of risk-free discount rate. Finally, we assumed recovery rates of 45% for all credits.

Table 9: Simulation Results – Loss Basket

	Whole Portfolio	0%-20% Loss Tranche		40%-100% Loss Tranche	
Correlation	Average # of defaults	PV of losses ($)	Breakeven spread (bps)	PV of losses ($)	Breakeven spread (bps)
0	1.11	51.94	1504	0.26	2
0.3	1.12	46.10	1282	1.76	12
0.5	1.13	41.88	1129	3.41	23
0.7	1.13	37.11	965	6.12	42
0.99	1.14	24.64	584	14.63	103

For the loss basket, shown in Table 9, the equity tranche (with a 0% attachment point and a 20% detachment point) absorbs more than one default up to a total loss of $100, the tranche notional. The equity tranche's breakeven spread is as high as 1504 bps when correlation is zero, but it drops to 584 bps when correlation is 99%. While the losses and breakeven spreads for the 0%-20% tranche decrease as correlation increases from 0% to 99%, for the senior (40%-100%) tranche, losses and breakeven spreads increase as correlation rises.

Table 10: Simulation Results – Nth-to-Default Basket

	Whole Portfolio	1st-to-Default		2nd-to-Default		3rd to 5th-to-Default	
Correlation	Average # of defaults	PV of losses ($)	Breakeven spread (bps)	PV of losses ($)	Breakeven spread (bps)	PV of losses ($)	Breakeven spread (bps)
0	1.11	38.41	1383	16.54	386	4.79	33
0.3	1.12	32.51	1041	16.61	398	6.04	78
0.5	1.13	28.49	849	16.37	395	15.58	111
0.7	1.13	24.22	671	15.75	381	20.94	152
0.99	1.14	14.09	337	12.89	303	33.99	261

As we can see in Table 10, tranche losses and spreads to the first-to-default decline as the correlation increases. The breakeven spread for the FTD tranche drops from 1383 bps to 337 bps as correlation increases from 0% to 99%. On the other hand, for the senior tranche (3rd to 5th-to-default), tranche losses and spreads increase from with correlation, with spread increasing from 33 bps to 261 bps.

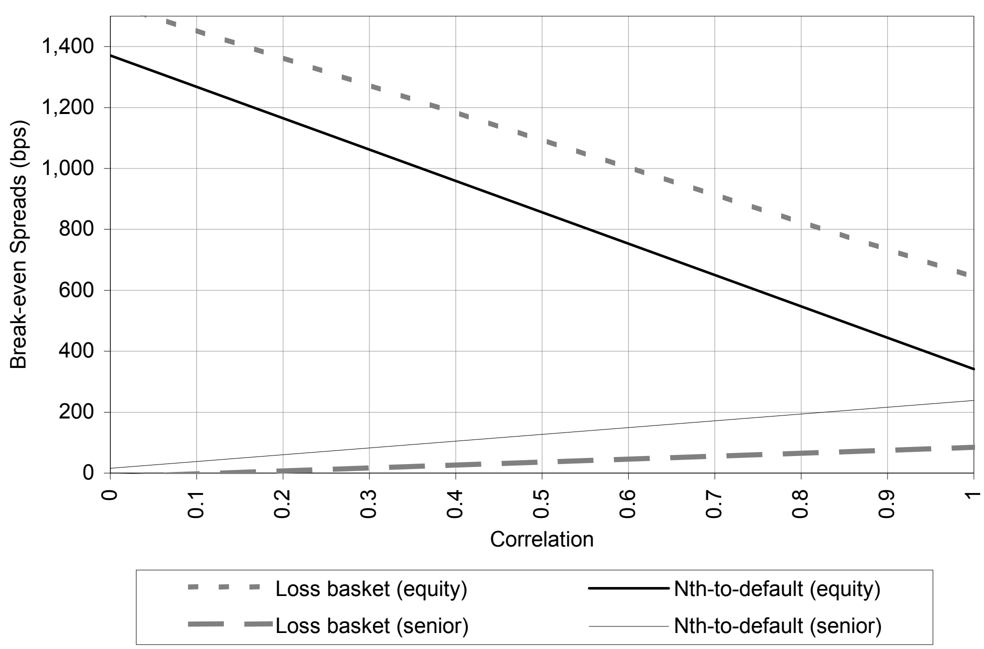
Hence, higher correlation benefits the value of the equity tranche (decreases the value of protection) and decreases the value of the senior tranche (increases the value of protection). On the other hand, the mezzanine tranche (2nd-to-default) is fairly insensitive to changes in correlation. Also note that the breakeven spreads for the first-, second-, and the senior (3-5th) tranches are very close (337 bps,

[11] Note that, although we do not show in the below, a correlation of 100% indicates that all reference entities default at the same time, resulting in a structure where there is only one entity, instead of five, in the portfolio. In such a case, the values of first-to-default and fifth-to-default baskets should be the same, since cash flows to the two tranches are identical.

303 bps, and 261 bps) when correlation is 99%. This is because, with the very high level of correlation, all five credits tend to default at the same time, reducing the differences between junior and senior tranches.

Graph 4 compares simulated spreads for equity and senior tranches for different correlation levels for the n^{th}-to-default basket and the percentage loss basket. In both loss tranches and n^{th}-to-default baskets, the breakeven spreads of the equity tranches decline as correlation increases, while the spreads of the senior tranches increase as correlation rises. Also we can see that the breakeven spread is higher for a loss tranche than an n^{th}-to-default basket in the equity tranche, while the reverse is true for the senior tranche.

Graph 4: Tranche Spreads and Correlation



Source: Nomura

C. More on Copula

A copula is a multivariate distribution function with uniformly distributed marginal distributions. A copula function, *C*, is defined where;

(a) There are uniform random variables, $U_1 \ldots U_n$, taking values in [0,1] such that function *C* is their joint distribution functions,

$$C\,(u_1, \ldots, u_n) = \Pr\,[U_1 < u_1, \ldots, U_n < u_n]$$

and

(b) Function *C* has uniform marginal distributions where:

$$C\,(1, \ldots, 1, u_i, 1, \ldots, 1) = u_i \text{ for all } i \le n,\ u_i \in [0,1]$$

For an *n*-dimensional copula, there exists an n-dimensional distribution, *F,* such that

$$C\,\{F_1(x_1), \ldots, F_n(x_n)\} = F(x_1, \ldots, x_n)$$

where $F_i(x_i)$ is a marginal distribution. It follows from

$$
\begin{aligned}
C(u_1, \ldots, u_n) &= \Pr[U_1 < u_1, \ldots, U_n < u_n] \\
&= \Pr[U_1 < F_1(x_1), \ldots, U_n < F_n(x_n)] \\
&= \Pr[F_1^{-1}(U_1) < x_1, \ldots, F_n^{-1}(U_n) < x_n] \\
&= \Pr[X_1 < x_1, \ldots, X_n < x_n] \\
&= F(x_1, \ldots, x_n).
\end{aligned}
$$

Sklar (1959)[12] showed the reverse, or that an n-dimensional distribution function F can be written in a form of copula

$$F(x_1, \ldots, x_n) = C\{F_1(x_1), \ldots, F_n(x_n)\}$$

If F_i is continuous, C is uniquely defined.

This result is significant for credit risk modeling because it means that for any multivariate distribution function, the marginal distribution and the dependency structure can be separated. In other words, the distributions of individual reference entities' default times (marginal distributions) are irrelevant to the dependency structure among them.

In a Gaussian copula model, for example, the copula function is a multivariate cumulative normal distribution function with a correlation matrix, Σ, containing $n(n-1)/2$ pair-wise correlation coefficients. In the copula function, each value of x_i in the joint distribution has a corresponding cumulative value, u_i. The u_i's characterize the dependency structure among the realized values from marginal distributions. Once we obtain u_i's, we can map them to the marginal distributions where each u_i is a cumulative probability in the marginal distribution for each reference entity's default time, τ_i.

— END —

[12] Sklar, A., *Fonctions de répartition à n dimensions et leurs marges*, Publ. Inst. Statist. Univ. Paris, 8, 229-231, 1959.

V. Recent Nomura Fixed Income Research

Fixed Income General Topics

- U.S. Fixed Income 2004 Mid-Year Outlook/Review (1 July 2004)
- Report from Arizona 2004: Coverage of Selected Sessions of the Winter Securitization Conferences (10 February 2004)
- U.S. Fixed Income 2004 Outlook/2003 Review (18 December 2003)

ABS

- ABS/MBS Disclosure Update #5: Reactions to the Comment Letters (4 August 2004)
- ABS/MBS Disclosure Update #4: Issues from ASF Sunset Seminar (13 May 2004)
- ABS/MBS Disclosure Update #3: Start Your Engines – Get Ready to Comment (10 May 2004)
- ABS/MBS Disclosure Update #2 (5 May 2004)
- ABS/MBS Disclosure Update (29 April 2004)

MBS

- Nomura GNMA Project Loan Prepayment Report - June 2004 Factors (10 June 2004)
- Monthly Update on FHA/VA Reperforming Mortgages: Historical Prepayment Speeds, Default Losses, and Total Returns (3 June 2004)
- Nomura GNMA Project Loan Prepayment Report – May 2004 Factors (14 May 2004)
- Monthly Update on FHA/VA Reperforming Mortgages: Historical Prepayment Speeds, Default Losses, and Total Returns (14 May 2004)
- GNMA Project Loan REMIC Factor Comparison (20 April 2004)

Strategy

- Value in Two-Tiered Index Bonds (TTIBs) July 30, 2004
- Commercial Real Estate Sector Update - Multifamily July 30, 2004
- Commercial Real Estate Sector Update - Retail July 29, 2004
- Commercial Real Estate Sector Update - Office July 22, 2004
- GNMA II Premiums Continue to Look Cheap July 22, 2004
- Trade Idea: A “Positive Carry” Flattener July 16, 2004
- FICO Scores: A Quick Refresher July 13, 2004
- MBS Market Check-up: Post FOMC Update July 9, 2004
- New Outlook: Post FOMC and Employment Number July 8, 2004
- CMBS: Loan Extensions – not a near term problem July 1, 2004
- Trade Recommendation: 10-Year AAA CMBS June 25, 2004
- CMO VADM Bonds Offer Excellent Extension Protection (17 June 2004)
- Partial Duration: A Portfolio Strategy Tool (10 June 2004)
- Corporate Bonds - A 30,000 Foot View (7 June 2004)
- Inflation Linked Bonds - An Emerging Sector (2 June 2004)
- Update on Terrorism Insurance (1 June 2004)
- MBS Check-up: Update & Thoughts on Extension (25 May 2004)

Corporates

- Corporate Weekly - For the week ended 25 June 2004 (28 June 2004)
- Corporate Weekly - For the week ended 18 June 2004 (21 June 2004)
- Corporate Weekly - For the week ended 11 June 2004 (14 June 2004)
- US Corporate Sector Review - July (4 August 2004)
- US Corporate Sector Review - June (7 July 2004)
- US Corporate Sector Review - May (6 June 2004)

NOMURA

NEW YORK	TOKYO	LONDON
Nomura Securities International 2 World Financial Center, Building B New York, NY 10281 (212) 667-9300	Nomura Securities Company 2-2-2, Otemachi, Chiyoda-Ku Tokyo, Japan 100-8130 81 3 3211 1811	Nomura International PLC Nomura House 1 St Martin's-le-grand London EC1A 4NP 44 207 521 2000

Nomura Fixed Income Research

New York

David P. Jacob	(212) 667 2255	International Head of Research
David Resler	(212) 667 2415	Head of U.S. Economic Research
Mark Adelson	(212) 667 2337	Securitization/ABS Research
John Dunlevy	(212) 667 9298	Structured Products Strategist
Arthur Q. Frank	(212) 667 1477	MBS Research
Louis (Trey) Ott	(212) 667 9521	Corporate Bond Research
Tain Schneider		Rate Strategist
Parul Jain		Deputy Chief Economist
Weimin Jin		Quantitative Analyst
Michiko Whetten	(212) 667 2338	Quantitative Credit Analyst
James Manzi	(212) 667 2231	Analyst
Elizabeth Hoyt	(212) 667 2339	Analyst
Edward Santevecchi	(212) 667 1314	Analyst
Tim Lu		Analyst
Jeremy Garfield		Analyst
Kumiko Kimura		Translator
Tomoko Nago-Kern		Translator

Tokyo

Nobuyuki Tsutsumi	81 3 3211 1811	ABS Research

London

John Higgins	44 207 521 2534	Head of Research - Europe

I, Mark Adelson, a research analyst employed by Nomura Securities International, Inc., hereby certify that all of the views expressed in this research report accurately reflect my personal views about any and all of the subject securities or issuers discussed herein. In addition, I hereby certify that no part of my compensation was, is, or will be, directly or indirectly related to the specific recommendations or views that I have expressed in this research report.